NASA SMALL SELF-CONTAINED PAYLOAD PROGRAM

GET AWAY SPECIAL

G-093

STRUCTURAL VERIFICATION DOCUMENT

4 August 1997

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LIST OF REFERENCED DOCUMENTS

MSFC-SPEC-522B	Design Criteria for Controlling Stress Corrosion Cracking, Materials and Processes Laboratory, MSFC, 1 July 1987.
MIL-HDBK-5F	Military Handbook: Metallic Materials and Elements for Aerospace Vehicle Structures, 1 November 1990.
GAS Experimenter's Guide	NASA GAS Experimenter's Guide to the STS Safety Review Process and Data Package Preparation, GSFC, Sept. 1993.
Roark and Young, 1982	Roark, Raymond J. and Warren C. Young, *Formulas for Stress and Strain*, Fifth Ed., New York: McGraw-Hill, 1982.
Shigley and Mischke, 1989	Shigley and Mischke, *Mechanical Engineering Design*, Fifth Ed., New York: McGraw-Hill, 1989.
Marks' Handbook, 1978	*Marks' Standard Handbook for Mechanical Engineers*, Eighth Ed., Edited by Baumeister, Avallone and Baumeister, New York: McGraw-Hill, 1978.

NOMENCLATURE

c	Point most remote from neutral axis
E	Modulus of elasticity in tension; average ratio of stress to stress below proportional limit
E_c	Modulus of elasticity in compression; average ratio of stress to strain below proportional limit
F_n	Normal force
F_S	Shear force
FS	Factor of Safety
f_1	Fundamental frequency
g	Gravitational acceleration (= 32.174 ft/s^2 = 386.088 in/s^2)
G	Modulus of rigidity (shear modulus)
I	Moment of inertia
l	Length of structural element, length of moment arm
L_S	Shear load
L_T	Tension load
M	Bending moment
MS	Margin of Safety
MSU	Margin of Safety Ultimate
P_{all}	Ultimate or yield stress/force for computing MS
P_{max}	Maximum observed stress/force for computing MS
r	Radius of structural element
R	Reaction force
R_s	Shear load (or stress)/allowable shear load (or stress) for computing MS
R_t	Tensile load (or stress)/allowable load (or stress) for computing MS
t	Thickness of structural element
V	Transverse shear
w	Uniform load (W/l) along structural element
W	Load on structural element
σ_{cy}	Allowable compressive yield stress at which permanent strain equals 0.002
σ_{max}	Maximum fiber stress
σ_{su}	Allowable ultimate stress in pure shear (this value represents the average shearing stress over the cross section)
σ_{tu}	Allowable tensile strength
σ_{ty}	Allowable tensile yield stress at which permanent strain equals 0.002
μ	Poisson’s ratio

ACRONYMS AND ABBREVIATIONS

BP	Bottom Plate
CCD	Charge Coupled Device
EMP	Experiment Mounting Plate
ESS	Equipment Support Structure
FAS	Fluid Absorbing Sweeper
GAS	Get Away Special
CG	Center of Gravity
GSFC	Goddard Space Flight Center
HP	Half Plate
MSFC	Marshall Space Flight Center
NASA	National Aeronautics and Space Administration
PRV	Pressure Relief Valve
PSL	Particulate-Seeded Liquid
psi	pounds per square inch
SCC	Stress Corrosion Cracking
SS	Stainless Steel
TP	Top Plate
VORTEX	VOrtex Ring Transit EXperiment

1.0 INTRODUCTION

This document contains the structural verification of the G-093 equipment support structure (ESS). This document shows that the G-093 ESS can withstand flight limit loads of 10 g in the *x*-, *y*-, and *z*-axes with an ultimate factor of safety greater than 2.0 and a yield factor of safety of greater than 1.5. The fundamental frequency of the ESS about any axis is also greater than 35 Hz.

2.0 DESCRIPTION OF STRUCTURE

The payload consists of several hardware components: a fluid test cell including a vortex ring generator, a laser illumination system, a digital imaging system, a computer based controller, batteries to power the payload, heaters to maintain proper temperature on cold-sensitive items, dc brushless fans to circulate airflow, and a sensor array to monitor the operation of the entire system. An illustration of the components as they would appear in the support structure is given in Figure 2.0-2a through 2.0-2e.

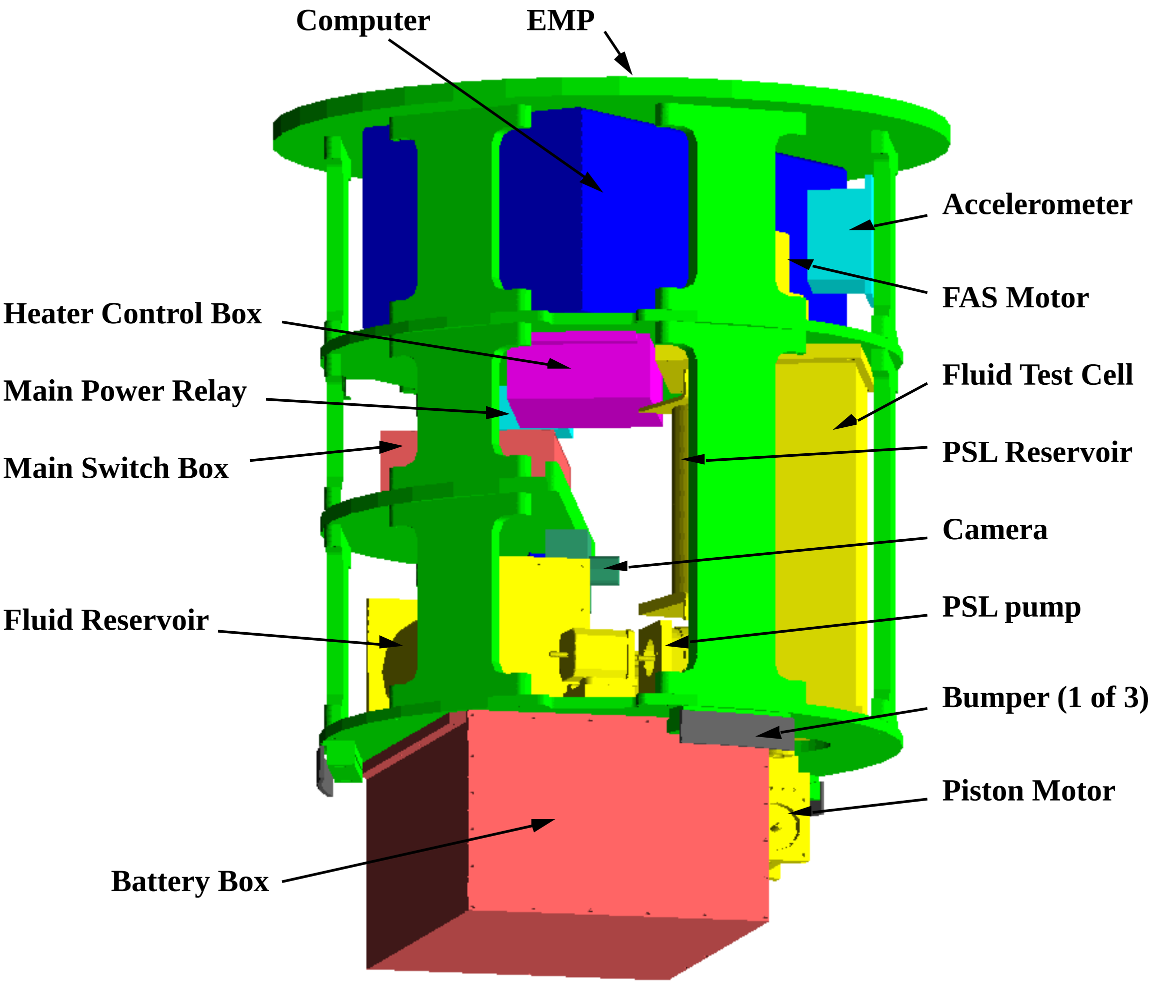

Figure 2.0-2a Illustration of G-093 components as they would appear in the support structure: view from one side

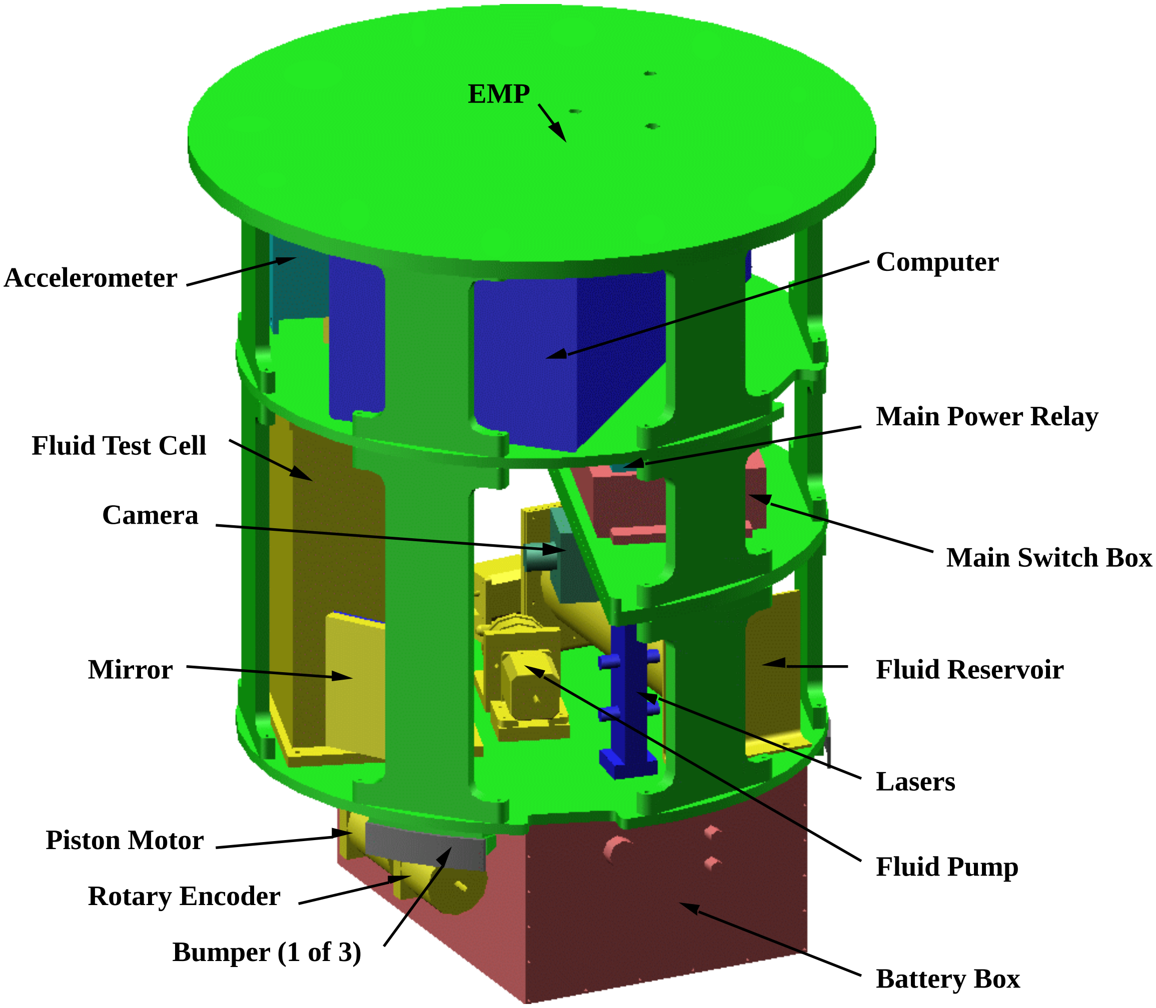

Figure 2.0-2b Illustration of G-093 components as they would appear in the support structure: opposite angle view

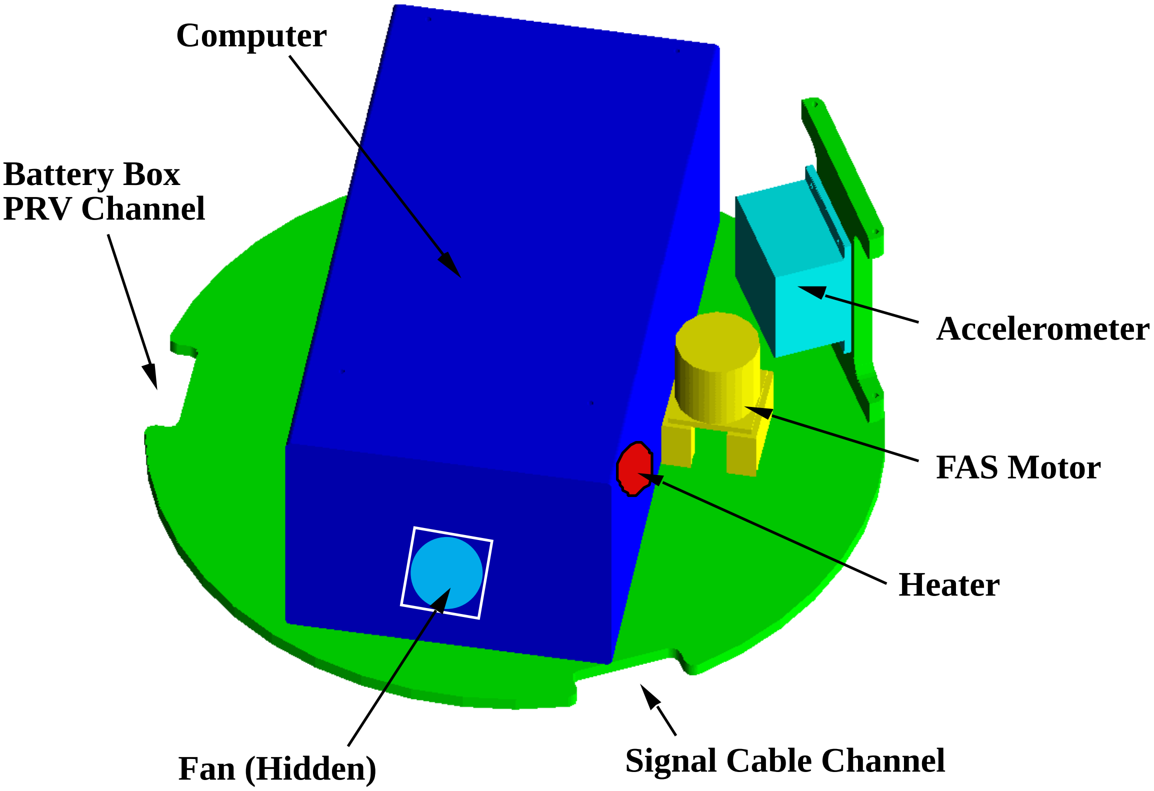

Figure 2.0-2c Illustration of G-093 components as they would appear in the support structure: top plate

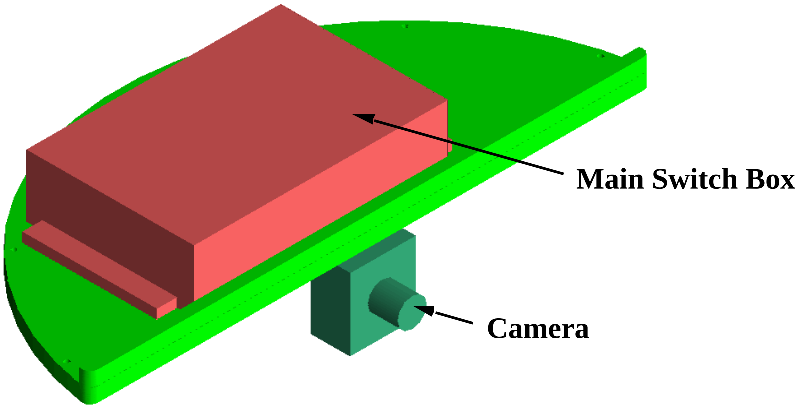

Figure 2.0-2d Illustration of G-093 components as they would appear in the support structure: half plate

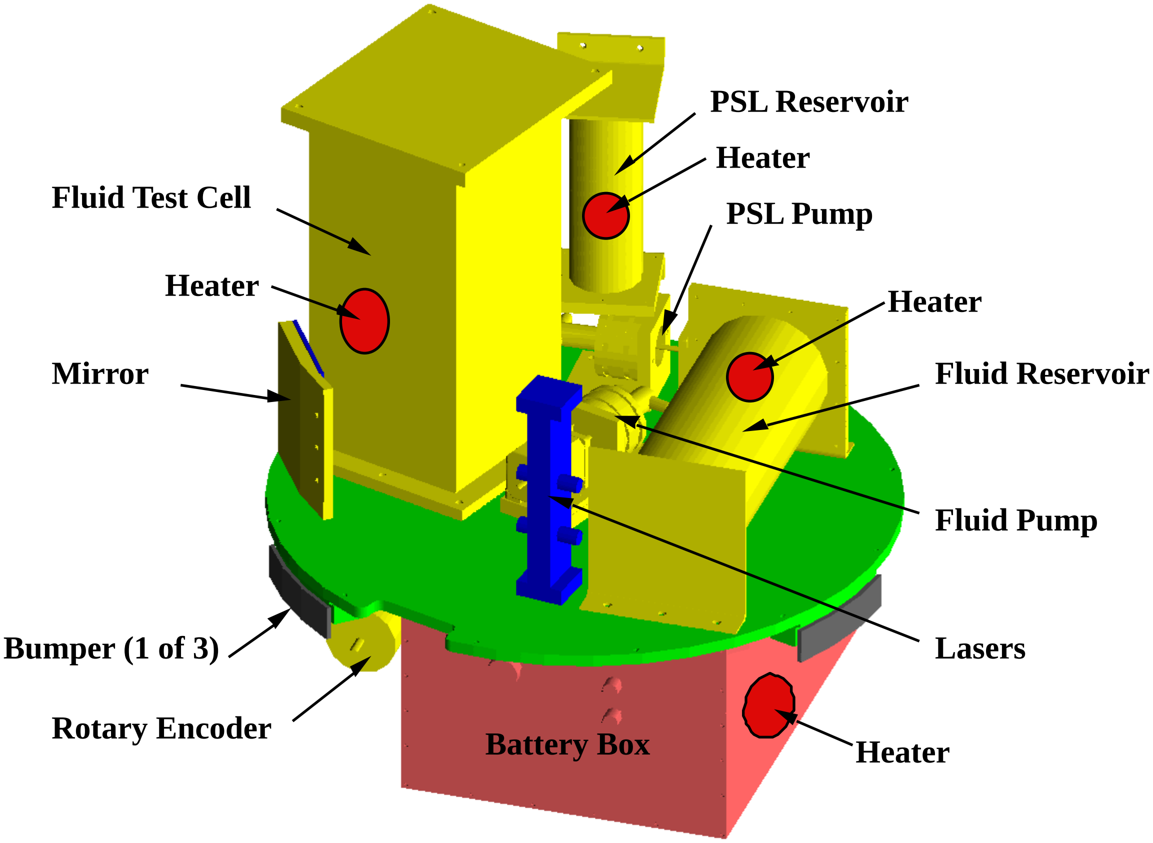

Figure 2.0-2e Illustration of G-093 components as they would appear in the support structure: bottom plate

2.1 SUPPORT STRUCTURE

The equipment support structure (ESS) to which the various G-093 components are mounted consists of two aluminum plates and one half plate each with a diameter of 19.5 in. (49.5 cm) and a thickness of 0.375 in. (0.953 cm). Six aluminum support beams are located symmetrically around the plates for stability. The total mass of the ESS is estimated at 50 lb. (23 kg) and is made entirely of 6061-T6 aluminum. All ESS fasteners used are UNBRAKO stainless steel (SS) socket head cap screws. A schematic of the support structure is shown in Figures 2.1-1a and b. All experiment components are fastened to the plates with 18-8 SS machine screws and using Loctite sealant. Table 2.1-1 gives a listing of all components and the fasteners used to secure them to the ESS.

Component	Location	Number of screws	Screw type 18-8 SS	Screw length (in.)
Computer	top of TP	4	10-32	0.750
Door motor and mount	top of TP	4	8-32	2.000
Accelerometer	on TP I-beam	4	8-32	0.750
Fan	bottom of TP	4	6-32	1.250
Heater control box	bottom of TP	4	10-32	0.500
Main power relay	bottom of TP	4	8-32	0.500
Main switch box	top of HP	4	10-32	0.500
Camera and mount	bottom of HP	4	10-32	1.750
Lasers and mount	between BP and HP	8	10-32	0.875
PSL reservoir and fluid	on middle I-beam	4	1/4-20	0.500
Mirror and mount	on middle I-beam	3	1/4-20	0.500
Fluid test cell	between TP and BP	14	1/4-20	0.875
Fluid reservoir and fluid	top of BP	6	1/4-20	0.500
PSL pump and mount	top of BP	4	1/4-20	1.125
PSL pump motor and mount	top of BP	4	6-32	1.125
Fluid pump and mount	top of BP	4	1/4-20	1.125
Fluid pump motor and mount	top of BP	4	6-32	1.125
Piston motor and mount	bottom of BP	3	1/4-20	1.750
Rotary encoder and mount	bottom of BP	3	1/4-20	1.750
Battery box	bottom of BP	12	1/4-20	0.875
Bumper base (3)	bottom of BP	2 (ea.)	10-32*	0.375
Bumper (3)	bottom of BP	3 (ea.)	1/4-28*	0.875

* UNBRAKO SS Screws Used

Table 2.1-1 Listing of all components attached to the ESS and the fasteners used to secure them

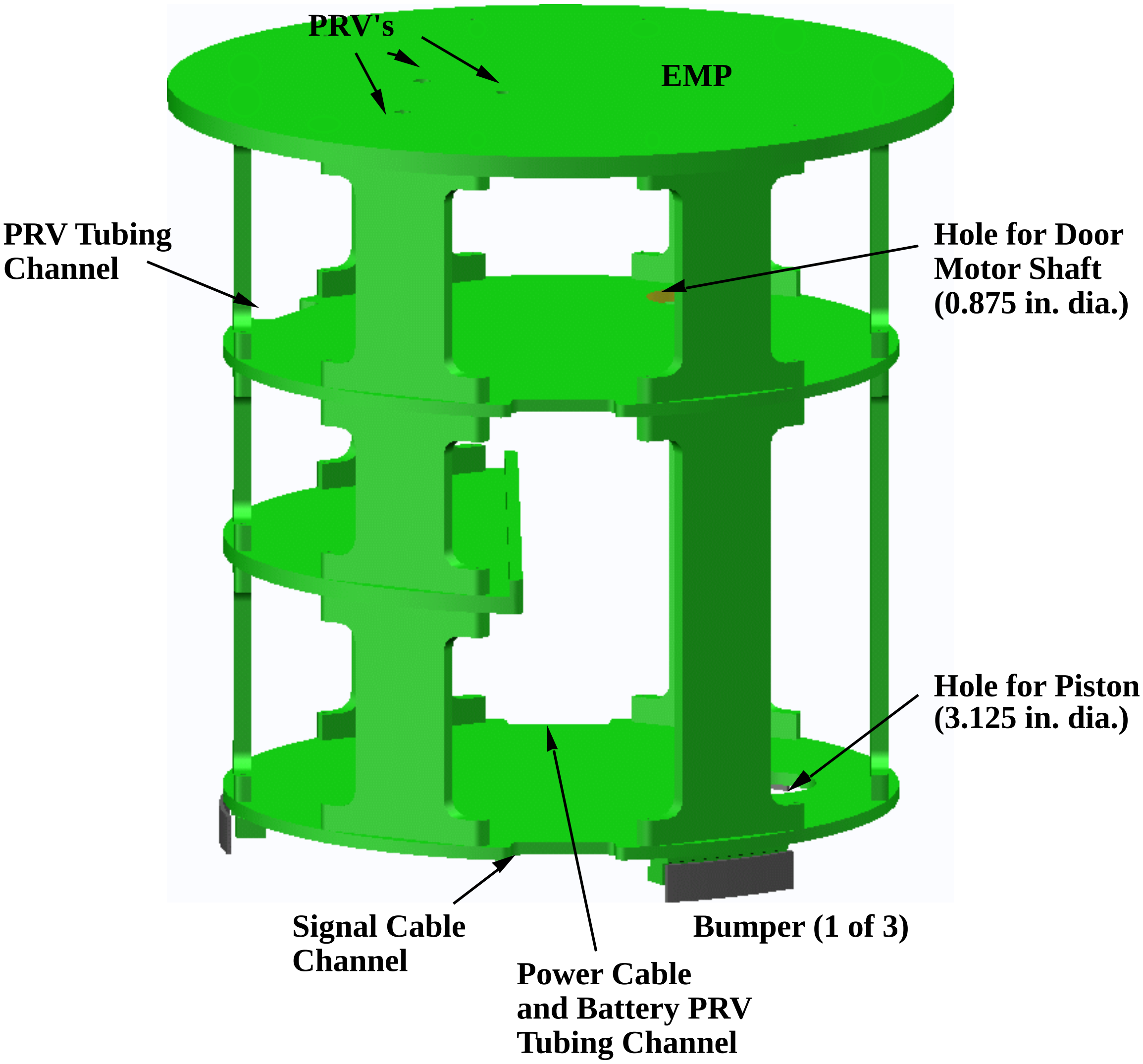

Figure 2.1-1a Drawing of the G-093 equipment support

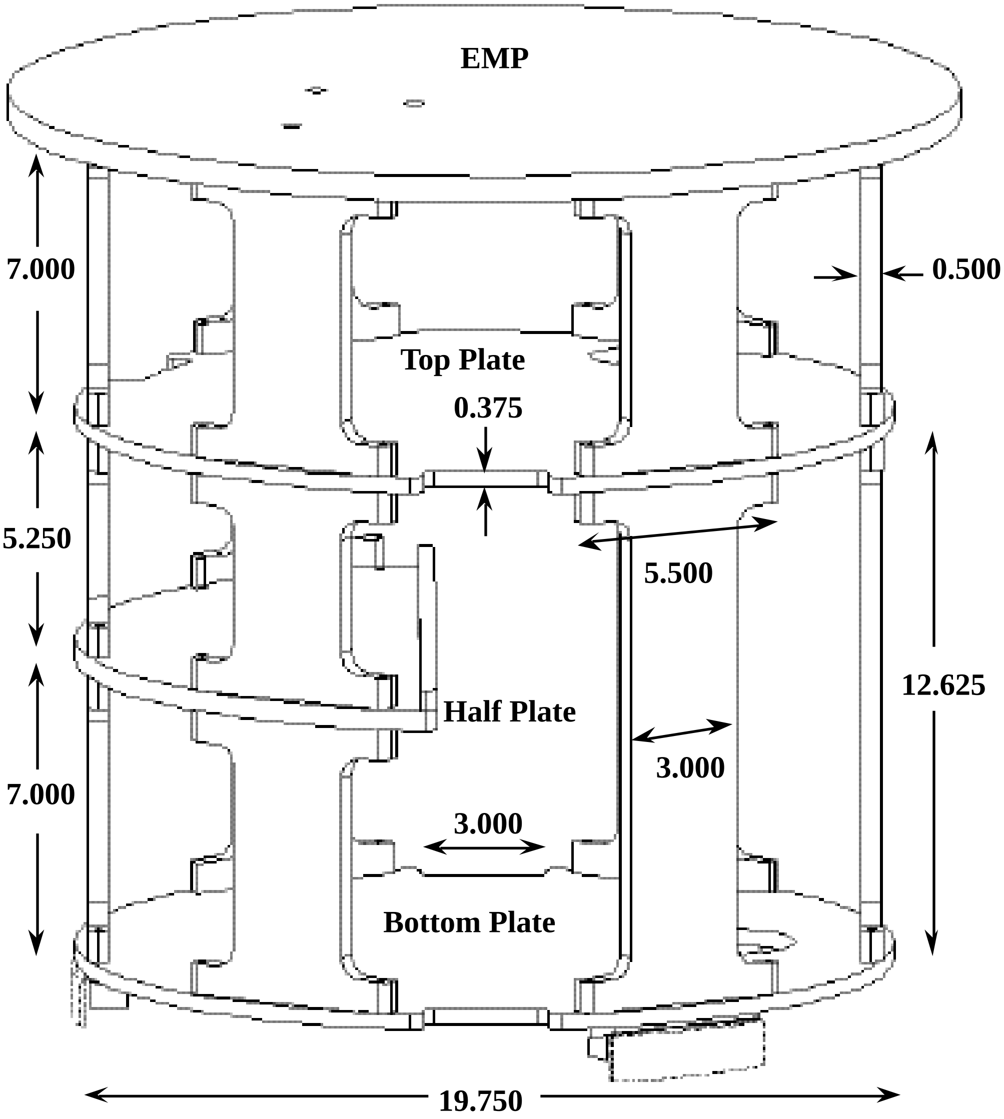

Figure 2.1-1b Dimensioned schematic of the G-093 equipment support structure

G-093 will be mounted to the NASA supplied EMP using twelve #10-32 UNBRAKO SS machine screws. These screws are fastened to the threaded EMP holes on the 19.0 in. diameter bolt circle. The bottom plate is attached to the posts using #10-32 UNBRAKO SS machine screws. The top plate and half plate are sandwiched between two posts with #10-32 UNBRAKO SS screws running through both posts and plate (see detail in Figure 2.1-1c).

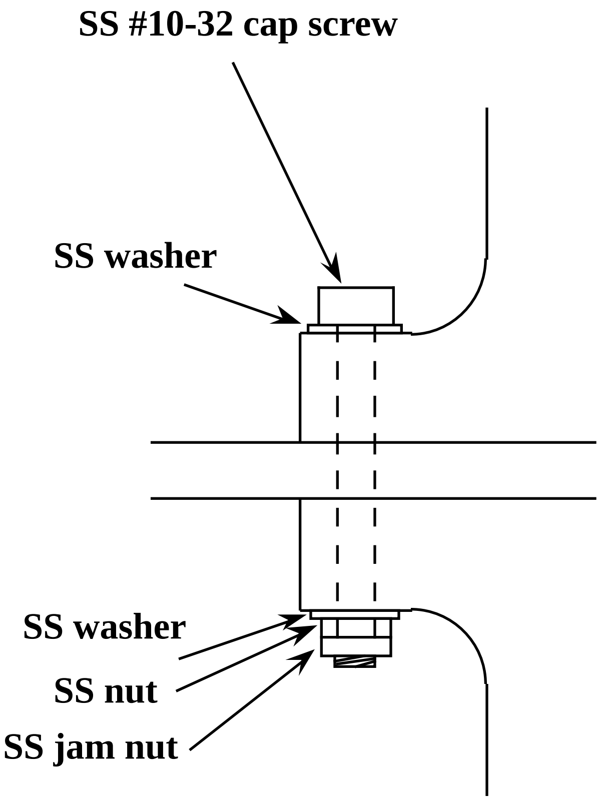

Figure 2.1-1c Detail of uprights sandwiching the plates

Three bumpers assemblies are affixed to the edge of the ESS bottom plate and contact the side of the GAS canister. Each bumper assembly provides approximately six square inches of surface contact area with the canister inner surface. Each bumper has a pad made of 0.125 in. thick Viton. Easy access to the bumpers for adjustment and installation is ensured by their location on the bottom side of the bottom plate of the ESS. The bumper bases are each attached with two #10-32 UNBRAKO SS screws, and the bumpers themselves are each attached with three 1/4-28 UNBRAKO SS screws.

3.0 MATERIAL PROPERTIES

The G-093 structural materials have been selected in accordance with MSFC-SPEC-522B to comply with Stress Corrosion Cracking (SCC) requirements. G-093 structural materials include 6061-T6 aluminum, 18-8 stainless steel, UNBRAKO SS socket head cap screws, and Viton rubber. Material properties which are important to the analysis are listed in the sections below.

3.1 6061-T6 ALUMINUM

The values for 6061-T6 aluminum have were obtained from Table 3.6.2.0(b_1) of MIL-HDBK-5F:

σ_{tu} = 42,000 psi
σ_{ty} = 35,000 psi
σ_{cy} = 35,000 psi
σ_{su} = 27,000 psi
E = 9,900,000 psi
E_c = 10,100,000 psi
G = 3,800,000 psi
μ = 0.33

3.2 18-8 STAINLESS STEEL

The values for 18-8 stainless steel were obtained from Table 2.7.1.0(b) of MIL-HDBK-5F (the values for annealed condition apply to AISI 304 grade stainless steel):

σ_{tu} = 73,000 psi
σ_{ty} = 26,000 psi
σ_{cy} = 23,000 psi
σ_{su} = 50,000 psi
E = 29,000,000 psi
E_c = 28,000,000 psi
G = 11,200,000 psi
μ = 0.27

3.3 UNBRAKO STAINLESS STEEL

The values for UNBRAKO stainless steel socket head cap screws (NAS1351) are :

σ_{tu} = 190,000 psi
σ_{ty} = 170,000 psi

3.4 VITON RUBBER

The allowable bearing stress for Viton is ≥ 750 psi.

3.5 SCREW PROPERTIES

The properties for screws used in this analysis are listed in Table 3.5-1.

Thread (UN) All Class 3A	10-32	10-32	1/4-20	1/4-28
Material	UNBRAKO SS	18-8 SS	18-8 SS	UNBRAKO SS
Tensile Strength, σ_u (psi)	190,000	75,000	75,000	190,000
Minimum Area, A_m (sq. in)	0.0175	0.0175	0.0269	0.0326
Stress Area, A_s (sq. in)	0.0199	0.0199	0.0317	0.0362
Tensile Strength, σ_{uc} (psi)	170,000	75,000	75,000	170,000
Bolt Ultimate Tensile Strength (lb)	3325	1312.5	2017.5	6194
Bolt Ultimate Shear Strength (lb)	2975	1312.5	2017.5	5542

Table 3.5-1 Screw properties used in this analysis

4.0 ASSUMPTIONS AND REFERENCE FRAME

The following assumptions were used in this analysis:

- Individual components modeled as concentrated point masses at the geometric center of the components.
- Loading results from the specified flight limit loads (10 g acceleration along the three axes of the payload) multiplied by the applicable factor of safety.
- No thermal gradients in the GAS payload (thermal effects on loading are negligible).
- All the loads are applied to the structure at the plates.
- The ESS supports are modeled as a rigid support at the EMP plate and a simple support at the bumpers location (bottom plate).
- For loads applied to the top and bottom plates, the ESS structure is modeled as a beam of uniform stiffness.
- The loads are distributed between the various support beams proportional to the beam stiffness in the load direction.
- Loads applied to the half plate are transferred to the reaction points using a modified equation that takes into account the nonuniform stiffness of the structure for those loads.

The coordinate reference frame for the analysis is given in Figure 4.0-1 below. The coordinate system is such that the z-axis is oriented through the center of the ESS with positive direction directed from the EMP to the bottom plate. The y-axis is oriented from the center of the ESS toward the half plate. The system is right-handed, so the x-axis is oriented out of the page in Figure 4.0-1 parallel to the edge of the half plate. The origin is located on the underside of the EMP as shown in Figure 4.0-1. Also shown in Figure 4.0-1 is the nomenclature for the ESS components used in the analysis.

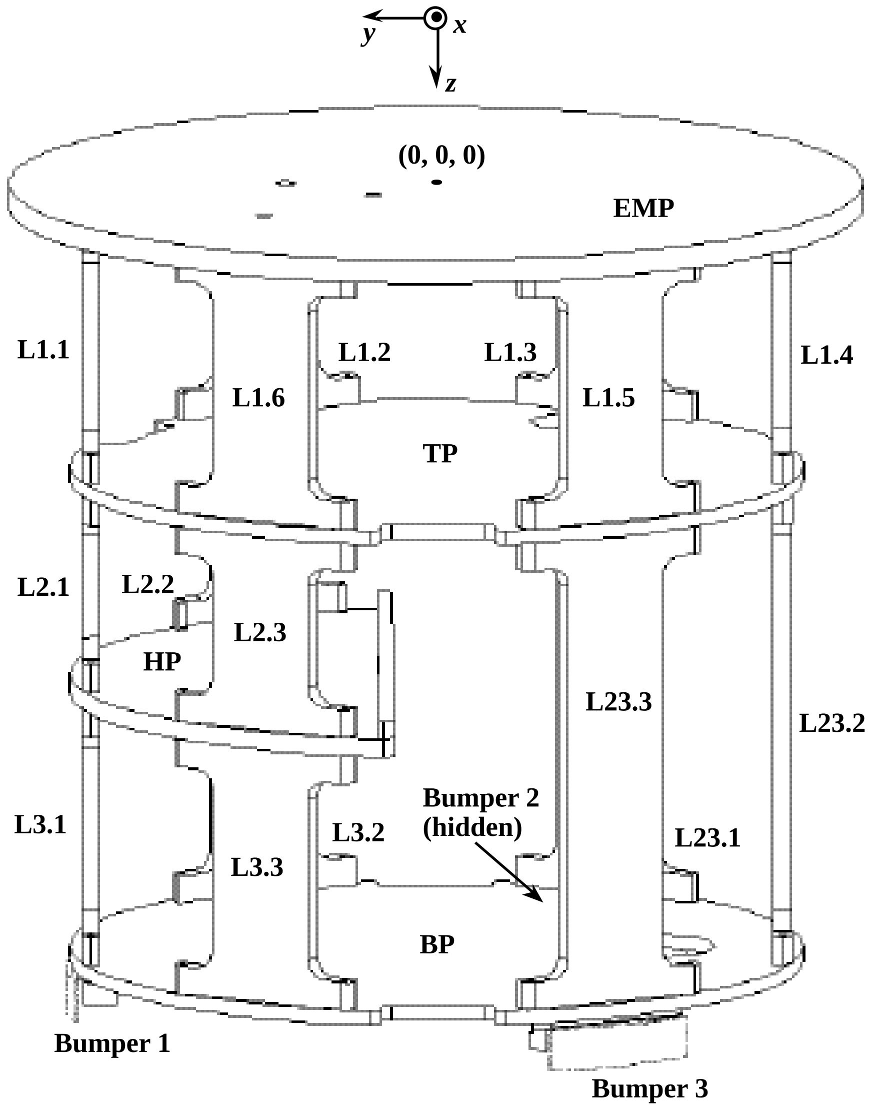

Figure 4.0-1 Schematic of the G-093 equipment support structure showing the reference frame used in the analysis

5.0 ANALYSIS

5.1 MARGINS OF SAFETY

The stability of G-093 was verified by analysis only. Following NASA requirements, an ultimate factor of safety of 2.0 is used in the analysis. Margins of safety are computed in the following manner. When computing a uni-axial load condition or a principle stress, the margin of safety, MS, is found via

$$\mathrm{MS} = \frac{P_{\mathrm{all}}}{P_{\mathrm{max}}\,\mathrm{FS}} - 1 \geq 0.0\,, \qquad (5.1\text{-}1)$$

where P_{all} is the ultimate or yield stress/force, P_{max} is the maximum observed stress/force, and FS is the factor of safety. The MS when combining shear and tension is found via

$$\text{MS} = \frac{1}{\sqrt{R_t^2 + R_s^2}} - 1 \geq 0.0, \qquad (5.1\text{-}2)$$

where R_t is the tensile load (or stress)/allowable load (or stress) and R_s is the shear load (or stress)/allowable shear load (or stress).

5.2 EQUIPMENT SUPPORT STRUCTURE ANALYSIS

The structural stability of the primary load bearing structure of G-093 (the ESS) was conducted using classical analysis tools and procedures. The results are summarized in Tables 5.2-1 to 5.2-6.

Table 5.2-1 lists the G-093 components, masses, attachment locations, and centers of gravity. For the analysis it is assumed that the load is a concentrated force equal to the mass multiplied by the acceleration applied at the plate indicated. The effect of the off-axis location of the center of mass is ignored.

Component	Load Location	Mass (lbs)	X_{CG}	Y_{CG}	Z_{CG}
Battery box (includes heater)	BP	55	0.472	1.424	23.375
Piston motor and rotary encoder	BP	4.774	-4.303	-4.33	23.065
Bottom plate	BP	10.56	0	0	20.1875
Fluid pump and motor	BP	2.42	6.442	-1.475	20.1875
PSL pump and motor	BP	2.42	2.655	-0.265	20.1875
Fluid reservoir (includes fluid & heater)	BP	9.02	1.94	4.06	20.1875
Lasers and mount	HP	0.88	-0.16	2.43	12.88
Mirror and mount	TP	1.408	-6.07	-6.058	7.175
CCD Camera and mount	HP	2.2	-0.025	1.328	12.88
PSL Reservoir (includes fluid & heater)	TP	4.4	6.222	-3.637	7.175
Test cell (includes FAS & heater)	TP	13.068	0	-5.4682	7.175
Test cell (includes FAS & heater)	BP	13.068	0	-5.4682	20.1875
Half plate	HP	10.01	0	4.767	12.88
Main switch box	HP	4.4	0	4.45	12.88
Main power relay	TP	1.1	-6.056	3.339	7.175
Heater control box	TP	2.2	3.117	2.756	7.175
Top plate	TP	30.36	0	0	7.175
Fan	TP	0.22	-6.495	-3.75	7.175
FAS motor and mount	TP	3.168	1.825	-5	7.175
Computer (includes heater)	TP	19.8	-0.3737	0.7668	7.175
Accelerometer	TP	0.88	0	-7.786	7.175

Table 5.2-1 Table listing the G-093 components, masses, attachment locations, and centers of gravity

For each component of the payload the reaction forces and moments at the supports were computed. The ESS was modeled as beam of constant stiffness with one end rigidly fixed at the EMP plate and the other end simply supported at the bumper location

(Table 3, case 1c, *Roark and Young*, 1982). With respect to Figure 5.2-1, the equations used to compute the reactive forces and moment at the supports are

$$R_A = \frac{W}{2l^3}(l-a)^2(2l+a) \tag{5.2-1}$$

$$R_B = \frac{Wa}{2l^3}\left(3l^2 - a^2\right), \tag{5.2-2}$$

$$\text{and}\, M_B = \frac{-Wa}{2l^2}\left(l^2 - a^2\right), \tag{5.2-3}$$

where W is the load (lbs); R_A and R_B are the vertical end reactions at the simply supported and the fixed end, respectively; M_A and M_B are the reaction end moments at the simply supported and the fixed end, respectively; and l is the beam length.

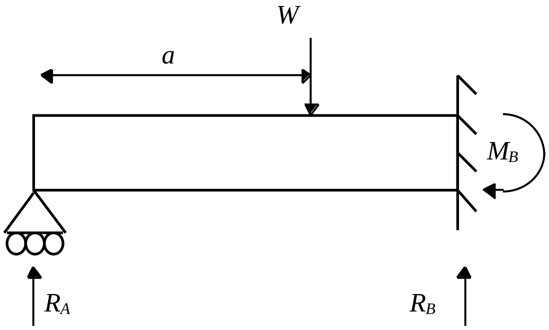

Figure 5.2-1 Beam model for structural analysis

The computed reaction forces and moment associated with each component of the payload are listed in Table 2.5-2 for a 10 g load in all directions. Also listed in Table 2.5-2 is the total reaction forces and moment for the payload under worst case loading. It is important to note that the present design results in a very large load at the bumpers.

Component	R_{bx} (lb)	R_{by} (lb)	R_{EMPx} (lb)	R_{EMPy} (lb)	M_{EMPx} (lb-in)	M_{EMPy} (lb-in)
Battery box (includes heater)	670.595	670.595	-120.595	-120.595	807.114	807.114
Piston motor and rotary encoder	57.139	57.139	-9.399	-9.399	63.091	63.091
Bottom plate	104.142	104.142	1.458	1.458	-9.899	-9.899
Fluid pump and motor	23.866	23.866	0.334	0.334	-2.269	-2.269
PSL pump and motor	23.866	23.866	0.334	0.334	-2.269	-2.269
Fluid reservoir (includes fluid & heater)	88.955	88.955	1.245	1.245	-8.456	-8.456
Lasers and mount	4.136	4.136	4.664	4.664	-29.078	-29.078
Mirror and mount	2.312	2.312	11.768	11.768	-53.925	-53.925
CCD Camera and mount	10.339	10.339	11.661	11.661	-72.696	-72.696
PSL Reservoir (includes fluid & heater)	7.224	7.224	36.776	36.776	-168.515	-168.515
Test cell (includes FAS & heater)	21.455	21.455	109.225	109.225	-500.490	-500.490
Test cell (includes FAS & heater)	128.876	128.876	1.804	1.804	-12.250	-12.250
Half plate	47.044	47.044	53.056	53.056	-330.767	-330.767
Main switch box	20.679	20.679	23.321	23.321	-145.392	-145.392
Main power relay	1.806	1.806	9.194	9.194	-42.129	-42.129
Heater control box	3.612	3.612	18.388	18.388	-84.258	-84.258
Top plate	49.844	49.844	253.756	253.756	-1162.755	-1162.755
Fan	0.361	0.361	1.839	1.839	-8.426	-8.426
FAS motor and mount	5.201	5.201	26.479	26.479	-121.331	-121.331
Computer (includes heater)	32.507	32.507	165.493	165.493	-758.319	-758.319
Accelerometer	1.445	1.445	7.355	7.355	-33.703	-33.703
Total	1305.404	1305.404	608.156	608.156	-2676.722	-2676.722

Table 5.2-2 Reactive force and moments at the supports for a load of 10 g applied simultaneously on the x and y axes

The bending moment and shear force acting on various ESS elements are determined from the load distribution and the reactive forces and moment at the supports. The equation used to compute the transverse shear, V, is

$$V = R_A - W\langle x - a\rangle^0, \tag{5.2-4}$$

and the bending moment, M, is

$$M = M_A + R_A x - W\langle x - a\rangle, \tag{5.2-5}$$

where the quantity $\langle x - a\rangle^0$ is defined as the unit step function and $\langle x - a\rangle = (x - a)\langle x - a\rangle^0$. Note that the shear force is constant between the plates where the external forces act, and the bending moment varies linearly between the plates. It follows that the maximum bending moment will be found at one of the plates or the EMP plate. Table 5.2-3 lists the shear force at the plates. The total shear force is listed at the bottom.

Component	$V_{\mathrm{EMP}x}$ (lb)	$V_{\mathrm{EMP}y}$ (lb)	$V_{\mathrm{TP}x}$ (lb)	$V_{\mathrm{TP}y}$ (lb)	$V_{\mathrm{HP}x}$ (lb)	$V_{\mathrm{HP}y}$ (lb)	$V_{\mathrm{BP}x}$ (lb)	$V_{\mathrm{BP}x}$ (lb)
Battery box (includes heater)	-120.595	-120.595	-120.595	-120.595	-120.595	-120.595	550.000	550.000
Piston motor and rotary encoder	-9.399	-9.399	-9.399	-9.399	-9.399	-9.399	47.740	47.740
Bottom plate	1.458	1.458	1.458	1.458	1.458	1.458	0.000	0.000
Fluid pump and motor	0.334	0.334	0.334	0.334	0.334	0.334	0.000	0.000
PSL pump and motor	0.334	0.334	0.334	0.334	0.334	0.334	0.000	0.000
Fluid reservoir (includes fluid & heater)	1.245	1.245	1.245	1.245	1.245	1.245	0.000	0.000
Lasers and mount	4.664	4.664	4.664	4.664	-4.136	-4.136	0.000	0.000
Mirror and mount	11.768	11.768	-2.312	-2.312	-2.312	-2.312	0.000	0.000
CCD Camera and mount	11.661	11.661	11.661	11.661	-10.339	-10.339	0.000	0.000
PSL Reservoir (includes fluid & heater)	36.776	36.776	-7.224	-7.224	-7.224	-7.224	0.000	0.000
Test cell (includes FAS & heater)	109.225	109.225	-21.455	-21.455	-21.455	-21.455	0.000	0.000
Test cell (includes FAS & heater)	1.804	1.804	1.804	1.804	1.804	1.804	0.000	0.000
Half plate	53.056	53.056	53.056	53.056	-47.044	-47.044	0.000	0.000
Main switch box	23.321	23.321	23.321	23.321	-20.679	-20.679	0.000	0.000
Main power relay	9.194	9.194	-1.806	-1.806	-1.806	-1.806	0.000	0.000
Heater control box	18.388	18.388	-3.612	-3.612	-3.612	-3.612	0.000	0.000
Top plate	253.756	253.756	-49.844	-49.844	-49.844	-49.844	0.000	0.000
Fan	1.839	1.839	-0.361	-0.361	-0.361	-0.361	0.000	0.000
FAS motor and mount	26.479	26.479	-5.201	-5.201	-5.201	-5.201	0.000	0.000
Computer (includes heater)	165.493	165.493	-32.507	-32.507	-32.507	-32.507	0.000	0.000
Accelerometer	7.355	7.355	-1.445	-1.445	-1.445	-1.445	0.000	0.000
Total	608.157	608.157	-157.883	-157.883	-332.783	-332.783	597.740	597.740

Table 5.2-3 Shear force at the ESS plates

Table 5.2-4 lists the bending moment at the plates. The bending moment varies linearly between the plates. The total bending moment is listed at the bottom. The maximum value in the total row is the maximum bending moment in the ESS, and should give the maximum stress on the ESS due to bending.

Component	$M_{\mathrm{EMP}y}$ (lb-in)	$M_{\mathrm{EMP}y}$ (lb-in)	$M_{\mathrm{TP}x}$ (lb-in)	$M_{\mathrm{TP}y}$ (lb-in)	$M_{\mathrm{HP}x}$ (lb-in)	$M_{\mathrm{HP}y}$ (lb-in)	$M_{\mathrm{BP}x}$ (lb-in)	$M_{\mathrm{BP}x}$ (lb-in)
Battery box (includes heater)	807.114	807.114	-59.659	-59.659	-746.144	-746.144	0	0
Piston motor and rotary encoder	63.091	63.091	-4.467	-4.467	-57.972	-57.972	0	0
Bottom plate	-9.899	-9.899	0.578	0.578	8.875	8.875	0	0
Fluid pump and motor	-2.269	-2.269	0.132	0.132	2.034	2.034	0	0
PSL pump and motor	-2.269	-2.269	0.132	0.132	2.034	2.034	0	0
Fluid reservoir (includes fluid & heater)	-8.456	-8.456	0.493	0.493	7.581	7.581	0	0
Lasers and mount	-29.078	-29.078	4.446	4.446	30.997	30.997	0	0
Mirror and mount	-53.925	-53.925	30.660	30.660	17.326	17.326	0	0
CCD Camera and mount	-72.696	-72.696	11.115	11.115	77.493	77.493	0	0
PSL Reservoir (includes fluid & heater)	-168.515	-168.515	95.814	95.814	54.142	54.142	0	0
Test cell (includes FAS & heater)	-500.490	-500.490	284.567	284.567	160.803	160.803	0	0
Test cell (includes FAS & heater)	-12.250	-12.250	0.715	0.715	10.983	10.983	0	0
Half plate	-330.767	-330.767	50.573	50.573	352.595	352.595	0	0
Main switch box	-145.392	-145.392	22.230	22.230	154.987	154.987	0	0
Main power relay	-42.129	-42.129	23.953	23.953	13.536	13.536	0	0
Heater control box	-84.258	-84.258	47.907	47.907	27.071	27.071	0	0
Top plate	-1162.755	-1162.755	661.115	661.115	373.582	373.582	0	0
Fan	-8.426	-8.426	4.791	4.791	2.707	2.707	0	0
FAS motor and mount	-121.331	-121.331	68.986	68.986	38.982	38.982	0	0
Computer (includes heater)	-758.319	-758.319	431.162	431.162	243.640	243.640	0	0
Accelerometer	-33.703	-33.703	19.163	19.163	10.828	10.828	0	0
Total	-2676.721	-2676.721	1694.41	1694.41	786.080	786.080	0.000	0.000

Table 5.2-4 Bending moment at the ESS plates

Other loading conditions (*i.e.*, ±10 g, ±10 g, ±10 g) result in the same absolute values of moments listed in Table 5.2-4 and of the shear values listed in Table 5.2-3.

The maximum fiber stress, $\sigma_{\max}$, is obtained by first determining the bending moment acting on each beam. The moment distribution is computed using

$$M_{xi} = M_x \frac{I_{xi}}{\sum I_{xi}}\,, \tag{5.2-6}$$

$$M_{yi} = M_y \frac{I_{yi}}{\sum I_{yi}}\,, \tag{5.2-7}$$

where M_{xi} and M_{yi} are the x and y bending moment acting on beam I, I_{xi} and I_{yi} are the x and y moments of inertia of beam I. With reference to Figure 5.2-2, the moments of inertia depend on the orientation of the beams and can be determined using the following equations (Chapter 5, Table 6, Mark's Standard Handbook, 1978):

$$I = \frac{bh}{12}\left(h^2 \cos^2 a + b^2 \sin^2 a\right), \tag{5.2-8}$$

$$I/c = \frac{bh}{6}\left(\frac{h^2 \cos^2 a + b^2 \sin^2 a}{h\cos a + b\sin a}\right), \tag{5.2-9}$$

where I/c is the section modulus. The moment of inertia and section modulus have been calculated for all the relevant beam orientations and are listed in Table 5.2-5. The total moment of inertia $I_x = I_y = 3.203$ in^4.

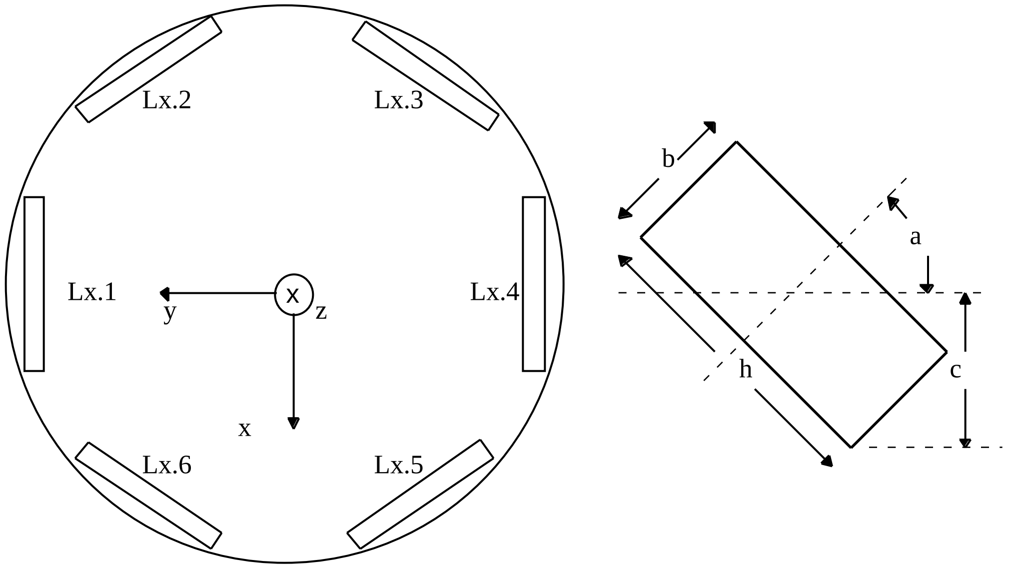

Figure 5.2-2 Key for calculating moments of inertia of the ESS beams

Moments of Inertia			Moment Inertia	Section Modulus
h (in)	*b* (in)	*a* (degrees)	I (in^4)	I/c (in^3)
2.92	0.50	0	3.0417E-02	1.2167E-01
2.92	0.50	30	2.8216E-01	2.9810E-01
2.92	0.50	60	7.8564E-01	5.6545E-01
2.92	0.50	90	1.0374E+00	7.1053E-01

Table 5.2-5 Table of moment of inertia and section modulus for relevant orientations of the beam cross sections for the ESS

For a beam in the ESS the maximum fiber stress, $\sigma_{\max,i}$, is found at the point or points most remote from the neutral axis and is given by

$$\sigma_{\max,i} = \frac{M_i}{(I/c)_i}, \tag{5.2-10}$$

the values of section modulus I/c are also listed in Table 5.2-5.

In addition to the contribution due to the bending moment, the normal load of 10 g in the z direction contributes to the maximum tension stress. The contribution due to the axial force is $\sigma = F_z$/total beam cross-sectional area. Table 5.2-6 lists in the beams-loading-tension-stress column the total maximum tension stress on each beam.

The shear load is determined as the shear force listed in Table 5.2-3 divided by the total cross-sectional area. The values are listed in the beams-loading-shear-load column of Table 5.2-6. The resulting MS's computed using Equation (5.1-2) are listed in the next column.

The moments and shear force at the plates are used to compute the load on the top and bottom fasteners of each beam. It is assumed that the moment acting on each beam at the plate is produced by loads acting on the fasteners. Also, the shear load on the beams and external loads are used to compute the shear load on the fasteners. These values are listed in Table 5.2-6 along with their respective margins of safety.

Beam	Beams Loading			Top Fasteners			Bottom Fasteners		
	Tension stress	Shear load	MSU	Tension load	Shear load	MSU	Tension load	Shear load	MSU
Beam L1.1	1647.3040	98.1808	11.6937	247.5911	71.6720	5.3887	274.2291	74.0478	4.8039
Beam L1.2	2170.3034	98.1808	8.6522	343.4495	71.6720	3.7141	297.5740	74.0478	4.3826
Beam L1.3	2170.3034	98.1808	8.6522	343.4495	71.6720	3.7141	297.5740	74.0478	4.3826
Beam L1.4	1647.3040	98.1808	11.6937	247.5911	71.6720	5.3887	274.2291	74.0478	4.8039
Beam L1.5	2170.3034	98.1808	8.6522	343.4495	71.6720	3.7141	297.5740	74.0478	4.3826
Beam L1.6	2170.3034	98.1808	8.6522	343.4495	71.6720	3.7141	297.5740	74.0478	4.3826
Beam L2.1	1057.0342	25.4887	18.8529	274.2291	74.0478	4.8039	157.2270	86.8177	7.9983
Beam L2.2	1388.1009	25.4887	14.1224	297.5740	74.0478	4.3826	168.2042	86.8177	7.5614
Beam L2.3	1388.1009	25.4887	14.1224	297.5740	74.0478	4.3826	168.2042	86.8177	7.5614
Beam L3.1	521.4181	25.4887	39.1588	157.2270	86.8177	7.9983	111.0297	39.2189	12.9274
Beam L3.2	772.1983	25.4887	26.1593	168.2042	86.8177	7.5614	111.0297	39.2189	12.9274
Beam L3.3	772.1983	25.4887	26.1593	168.2042	86.8177	7.5614	111.0297	39.2189	12.9274
Beam L23.1	1388.1009	25.4887	14.1224	297.5740	74.0478	4.3826	111.0297	39.2189	12.9274
Beam L23.2	1057.0342	25.4887	18.8529	274.2291	74.0478	4.8039	111.0297	39.2189	12.9274
Beam L23.3	1388.1009	25.4887	14.1224	297.5740	74.0478	4.3826	111.0297	39.2189	12.9274

Table 5.2-6 Stresses on the ESS structure and loads on the ESS fasteners for simultaneous 10 g loading

The margin of safety of the plates due to all the loads is determined using a simplified conservative model of the shear stress acting on the plate. The bottom plate is the more heavily loaded plate of the ESS and consequently we limit the analysis to the bottom plate. The bottom plate is modeled as a circular plate with edge simply supported (Chapter 5, Figure 69, case 6, and Table 19: Marks' Standard Handbook, 1978). With respect to Figure 5.2-3, the maximum stress for a concentrated load P is

$$\sigma_{\max} = k\frac{P}{t^2}, \qquad (5.2\text{-}11)$$

where the factor $k = 1.48$ is for the conservative case of $R/r = 2$. For a 10 g axial load, $P = 972$ lbs, and plate thickness $t = 0.375$ in gives a $\sigma_{\max} = 10236$ psi. The maximum shear stress is estimated as $\sigma_s = V_{\max}/A = 1837$ lbs/70 in^2 = 25 psi. This gives a MS = 1.05 for the bottom plate.

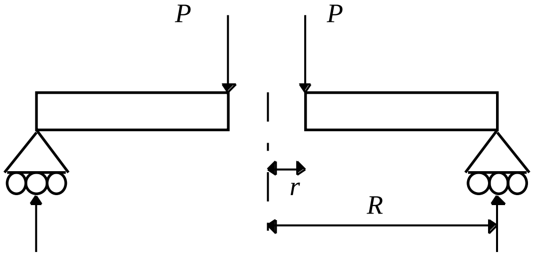

Figure 5.2-3 Circular plate model for structural analysis

A simple estimate of the buckling load for a rectangular column can be made using Euler's column formula (Chapter 5: Marks' Standard Handbook, 1978)

$$P_{cr} = n\pi^2 \frac{EI}{l^2}, \qquad (5.2\text{-}11)$$

where n is a coefficient to account for end conditions. A conservative value of $n = 1$ which applied to a column pivoted at both ends yields a

$$P_{cr} = \pi^2 \frac{(10{,}000{,}000\text{psi})3.203\text{in}^4}{23.375\text{in}^2} = 578{,}566\text{lb}.$$

This value gives a margin of safety for buckling of the ESS as MS > 200.

5.3 EXPERIMENT BOX FASTENER ANALYSIS

This section presents the maximum stress and shear analysis for all experiment boxes with weight greater than 5 lbs. All attachment fasteners for experiment boxes which weigh 5 lbs or more must be analyzed in detail for critical stresses and must exhibit positive margins of safety (GAS Experimenter's Guide, pg. B1-6). For G-093, this includes the computer, fluid test cell, fluid reservoir, and battery box.

The distribution of loads between fasteners is determined according to their stiffness. For all fasteners of the same size and stiffness, the fastener loads are given by the following equations:

$$L_{Ti} = \frac{M\, l_i}{\sum l_i^2} + \frac{F_n}{n}, \qquad (5.3\text{-}1)$$

where L_{Ti} is the tension load of the fastener, M is the moment of the load, l_i is the arm length from the bolt axis to the CG of the component normal to the moment axis, F_n is the normal force applied, and n is the number of bolts. The shear load acting on the fastener, L_{Si}, is given by

$$L_{Si} = \frac{F_S}{n}, \qquad (5.3\text{-}2)$$

where F_S is the shear force acting on the joint.

In the following sections, the tension load, shear load, and resulting margin of safety for worse case loading conditions are computed.

5.3.1 COMPUTER

The computer is attached to the top plate with 4 #10-32 18-8 SS screws and its total weight is 19.8 lbs. Figure 5.3.1-1 shows the fastener locations, and Table 5.3.1-1 gives the results of the calculations including the margin of safety developed under the following worst case loading conditions of 10 g acceleration:

M_x = 693 lb-in
M_y = 693 lb-in
F_n = 198 lb-in
F_s = 280 lb-in

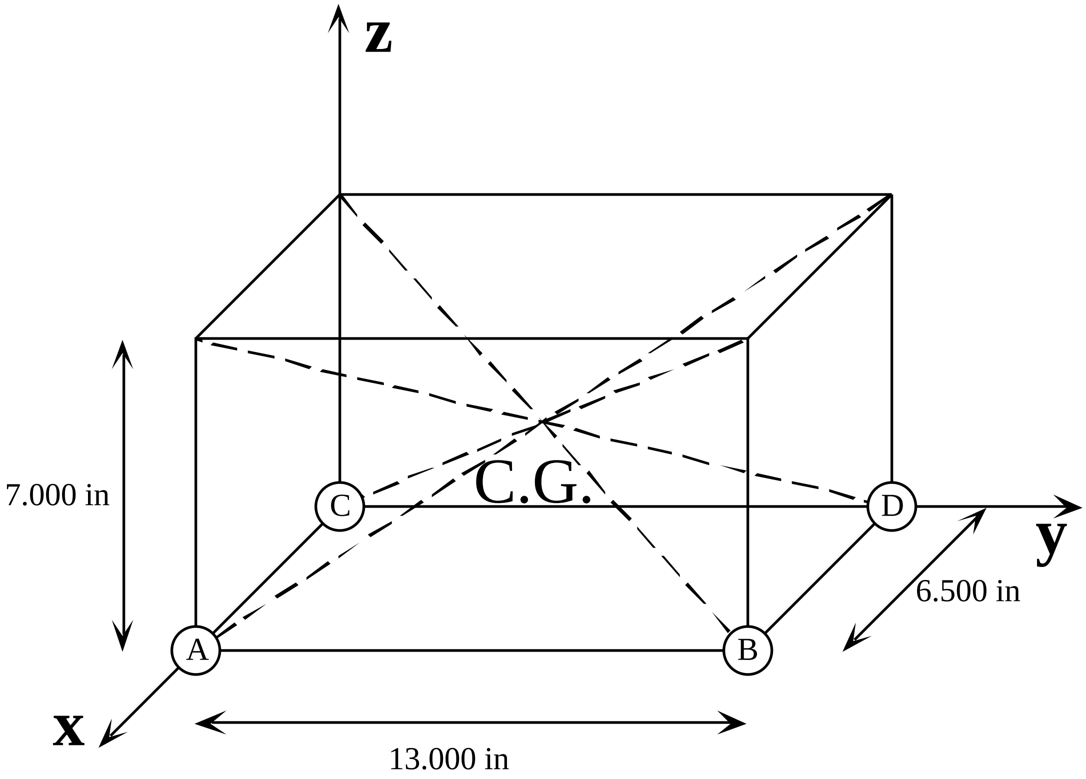

Figure 5.3.1-1 Location of fasteners (A through D) and CG for the computer

Fastener #	A	B	C	D
l_x	3.25	3.25	3.25	3.25
l_y	7.5	7.5	7.5	7.5
L_{Ti}	125.91	125.91	125.91	125.91
L_{Si}	70.00	70.00	70.00	70.00
MS	3.56	3.56	3.56	3.56

Table 5.3.1-1 Table of load distribution and margin of safety for the computer box fasteners

5.3.2 FLUID TEST CELL

To determine worst case loads on the bolts holding the fluid test cell, it is assumed that one end is fixed while the other is free. The analysis is the same for both the fasteners which connect it to the bottom plate and to the top plate, so only the case of connection to the bottom plate will be considered. Each end is connected via 7 1/4-20 18-8 SS screws and the total weight of the test cell is 26.1 lbs. Figure 5.3.2-1 shows the fastener locations, and Table 5.3.2-1 gives the results of the calculations including the margin of safety developed under the following worst case loading conditions of 10 g acceleration:

$$M_x = 1649.8 \text{ lb-in}$$
$$M_y = 1649.8 \text{ lb-in}$$
$$F_n = 261.4 \text{ lb-in}$$
$$F_s = 369.7 \text{ lb-in}$$

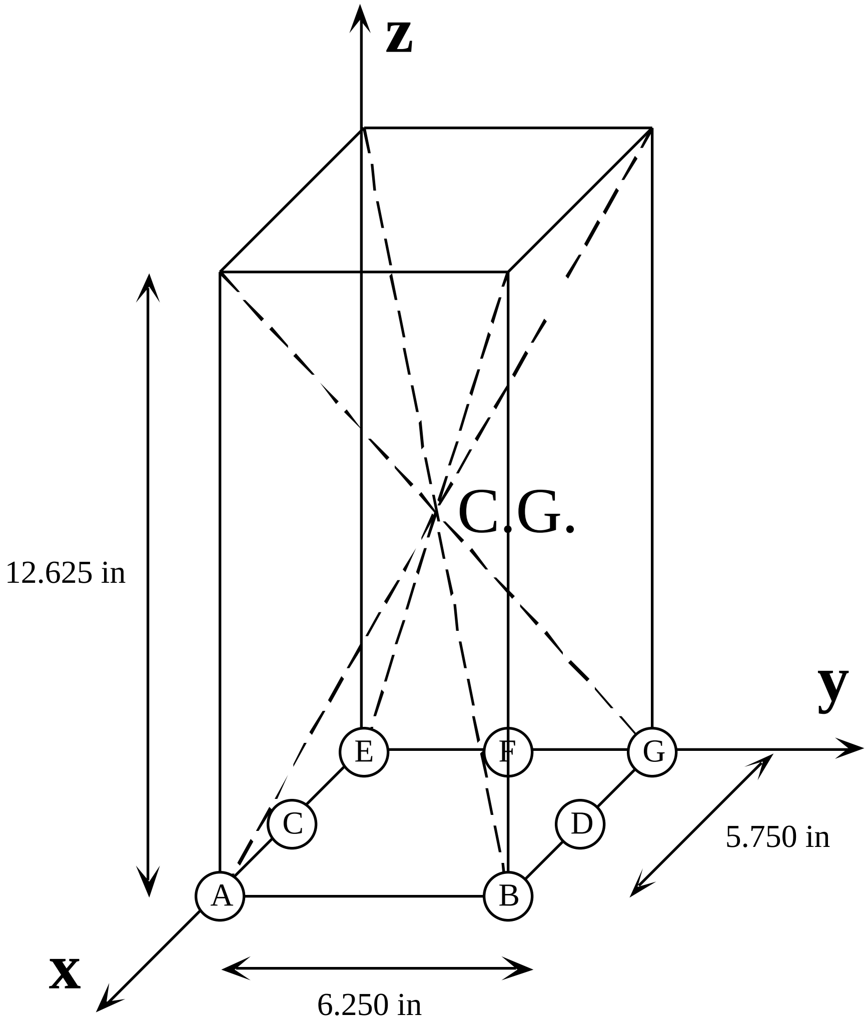

Figure 5.3.2-1 Location of fasteners (A through G) and CG for the fluid test cell

Fastener #	A	B	C	D	E	F	G
l_x	3.125	3.125	3.125	3.125	3.125	0	3.125
l_y	2.875	2.875	0	0	2.875	2.875	2.875
L_{Ti}	240.10	240.10	125.33	125.33	240.10	152.11	240.10
L_{Si}	52.80	52.80	52.80	52.80	52.80	52.80	52.80
MS	3.10	3.10	6.42	6.42	3.10	5.27	3.10

Table 5.3.2-1 Table of load distribution and margin of safety for the fluid test cell fasteners

5.3.3 FLUID RESERVOIR

The fluid reservoir is attached to the bottom plate with 6 1/4-20 18-8 SS screws and its total weight is 9.02 lbs. Figure 5.3.3-1 shows the fastener locations, and Table 5.3.3-1 gives the results of the calculations including the margin of safety developed under the following worst case loading conditions of 10 g acceleration:

$$M_x = 236.8 \text{ lb-in}$$
$$M_y = 236.8 \text{ lb-in}$$
$$F_n = 90.2 \text{ lb-in}$$
$$F_s = 127.6 \text{ lb-in}$$

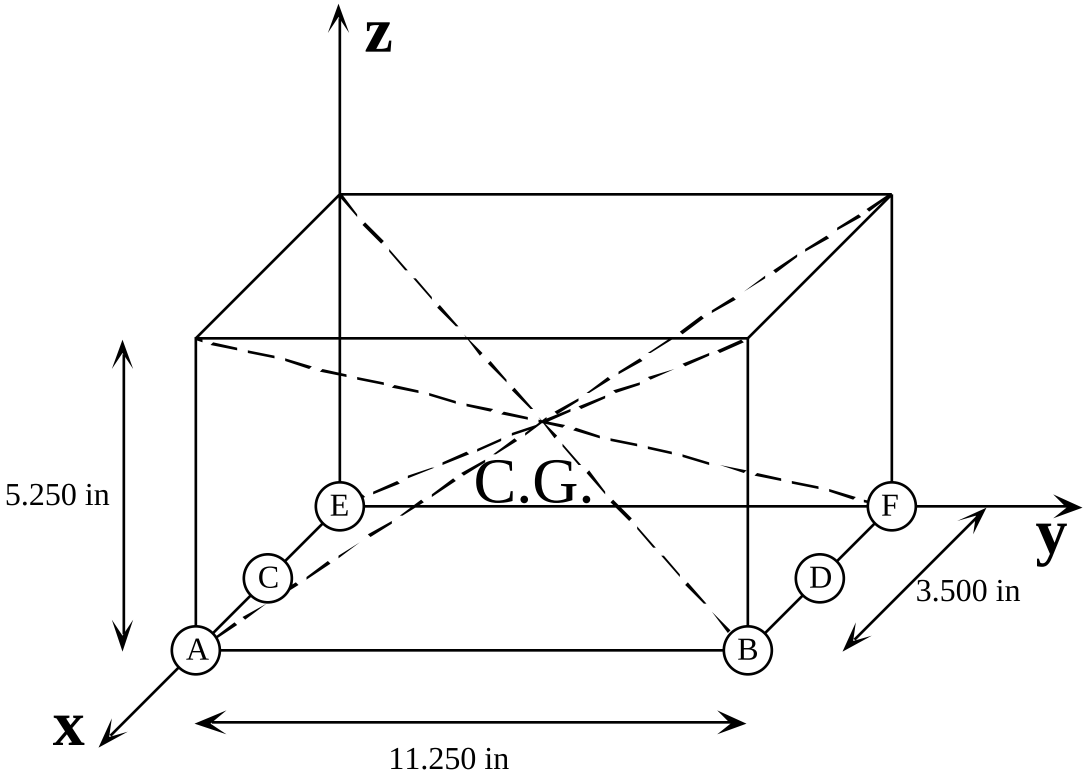

Figure 5.3.3-1 Location of fasteners (A through F) and CG for the fluid reservoir

Fastener #	A	B	C	D	E	F
l_x	1.625	1.625	0	0	1.625	1.625
l_y	5.625	5.625	5.625	5.625	5.625	5.625
L_{Ti}	58.48	58.48	22.05	22.05	58.48	58.48
L_{Si}	21.26	21.26	21.26	21.26	21.26	21.26
MS	15.21	15.21	31.93	31.93	15.21	15.21

Table 5.3.1-1 Table of load distribution and margin of safety for the fluid reservoir fasteners

5.3.4 BATTERY BOX

The battery box is attached to the bottom plate with 12 1/4-20 18-8 SS screws and its total weight is 55.0 lbs. Figure 5.3.4-1 shows the fastener locations, and Table 5.3.4-1 gives the results of the calculations including the margin of safety developed under the following worst case loading conditions of 10 g acceleration:

$$M_x = 5772.3 \text{ lb-in}$$
$$M_y = 5772.3 \text{ lb-in}$$
$$F_n = 550 \text{ lb-in}$$
$$F_s = 777.8 \text{ lb-in}$$

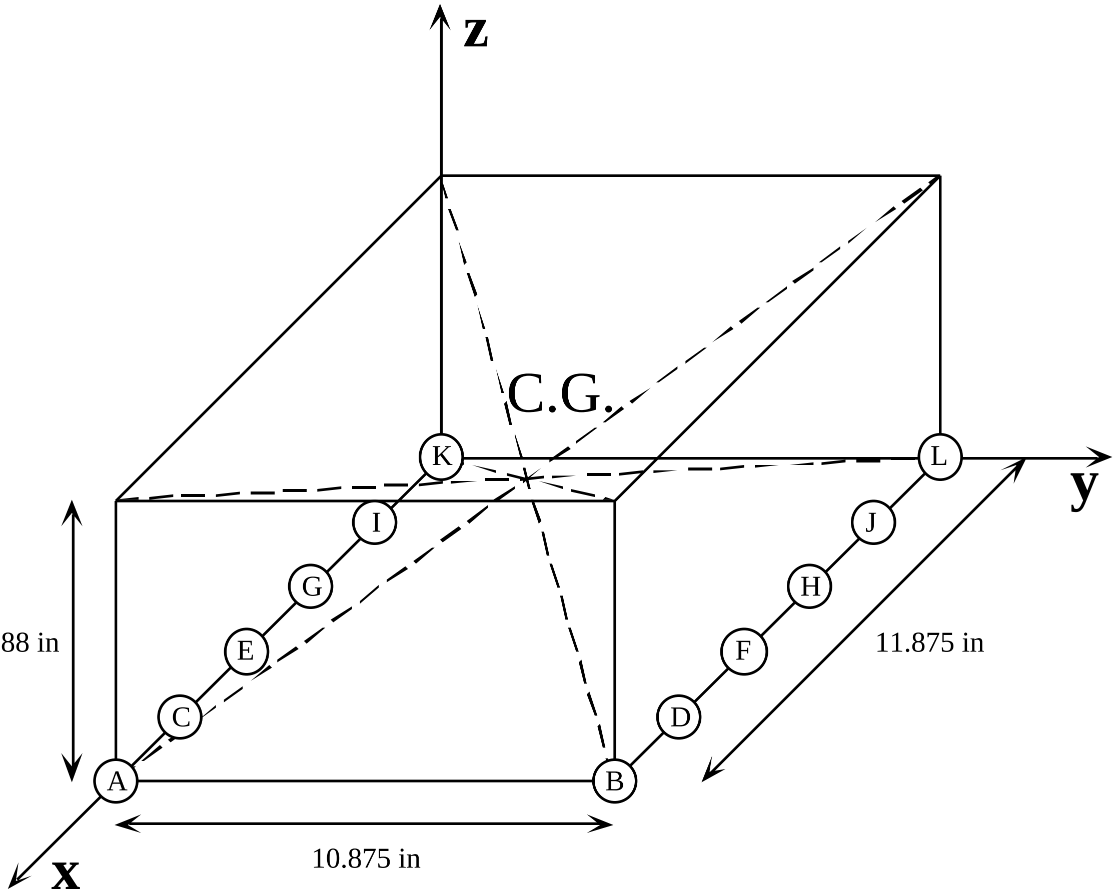

Figure 5.3.4-1 Location of fasteners (A through L) and CG for the battery box

Fastener #	A	B	C	D	E	F	G	H	I	J	K	L
l_x	5.9375	5.9375	3.5625	3.5625	1.1875	1.1875	1.1875	1.1875	3.5625	3.5625	5.9375	5.9375
l_y	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375	5.4375
L_{Ti}	307.90	307.90	238.46	238.46	169.02	169.02	169.02	169.02	238.46	238.46	307.90	307.90
L_{Si}	64.82	64.82	64.82	64.82	64.82	64.82	64.82	64.82	64.82	64.82	64.82	64.82
MS	2.21	2.21	3.08	3.08	4.57	4.57	4.57	4.57	3.08	3.08	2.21	2.21

Table 5.3.4-1 Table of load distribution and margin of safety for the battery box fasteners

5.4 BUMPER ANALYSIS

Three bumpers are located on the lower plate to limit lateral deflection of the G-093 ESS. The maximum bumper load is determined from the reactive load listed in Table 5.2-2 above. The three bumper arrangement results in a maximum load of

$$F_{\max} = (1305\text{lb})(1 + \tan 30°) = 2058\text{lb}\,.$$

The bumper mechanism is illustrated in Figure 5.4-1 below. For an 20° incline and three fasteners for each bumper the load on each fastener is

$$F_T = \frac{F_{\max}}{3\tan\,20°} = 1884\text{lb}$$

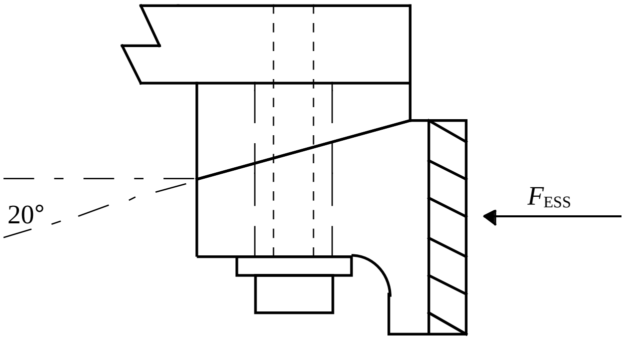

Figure 5.4-1 Side view of the G-093 bumper assembly

Three 1/4-28 UNBRAKO bolts support the load for each bumper. The ultimate strength (σ_u) for the bolts is 190,000 psi which gives a maximum load of 6194 lb. The margin of safety for the bumper fasteners is

$$\boxed{\text{MS} = \frac{(6194\text{lbs})}{2 \times 1884\text{lbs}} - 1 = 0.64}$$

Two 10-32 UNBRAKO bolts attach the ramp to the bottom plate. The ultimate single shear strength of these bolts is 2975 lbs which gives a margin of safety of

$$\boxed{\mathrm{MS} = \frac{(2975\mathrm{lbs})}{2(2058/\ 2)\mathrm{lbs}} - 1 = 0.446}$$

Each bumper has a Viton face with an area of 6 in^2. To calculate the bearing stress on the rubber bumper head, we begin by noting that Viton can withstand a bearing load of at least 750 psi. The bearing stress on the Viton face at each bumper will be 2058 lbs/6 in^2 = 343 psi, which gives a margin of safety of

$$\boxed{\mathrm{MS} = \frac{(750\mathrm{psi})}{2(343\mathrm{psi})} - 1 = 0.09}$$

5.5 FUNDAMENTAL FREQUENCY ANALYSIS

The resonant frequency of the loaded ESS can be determined analytically by finding the frequency of the first vibrational mode of the beam and the plates. We will examine the beam and plates under worst case mass loadings since this will give the lowest fundamental frequencies.

The fundamental frequency, f_1, of a uniform beam, one fixed end and one end hinged, with uniform load w including beam weight can be determined (Table 36, case 5a, *Roark and Young*, 1982)

$$f_1 = \frac{15.4}{2\pi}\sqrt{\frac{EIg}{wl^4}}\ , \qquad (5.5\text{-}1)$$

where E is the modulus of elasticity, I is the moment of inertia, and g is the gravitational acceleration. In our case, $w = W/l$ where W = 173.0 lbs and l = 20.1875 in which gives for the minimum resonant frequency of the beam mode

$$\boxed{f_1 = \frac{15.4}{2\pi}\sqrt{\frac{(9{,}900{,}000\mathrm{lbs/in}^2)(3.203\mathrm{in}^4)(386.0888\mathrm{in/s}^2)}{(8.57\mathrm{lbs/in})(20.1875\mathrm{in})^4}} = 227\mathrm{Hz}}.$$

The fundamental frequency, f_1, of a circular flat plate of uniform thickness t and radius r, edge free, with uniform load w per unit length including own weight can be determined approximately as (Table 36, case 11a, *Roark and Young*, 1982)

$$f_1 = \frac{4.99}{2\pi}\sqrt{\frac{Dg}{wr^4}}\ , \qquad (5.5\text{-}2)$$

where $D = Et^3/10.92$. The plate with the lowest fundamental frequency will be the bottom plate since it is loaded most heavily. For this case, the following numbers apply: E = 10,000,000 psi, t = 0.375 in., W = 91.2 lbs, where $w = W/(\pi r^2)$ giving the fundamental frequency of the loaded bottom plate as

$$\boxed{f_1 = \frac{4.99}{2\pi}\sqrt{\frac{\left(10{,}000{,}000\,\mathrm{lbs/in}^2\right)\left(0.375\,\mathrm{in}\right)^3}{10.92}\,\frac{\left(386.0888\,\mathrm{in/s}^2\right)}{\left[\left(91.2\,\mathrm{lbs}\right)\Big/\left(\pi\left(9.875\,\mathrm{in}\right)^2\right)\right]\left(9.875\,\mathrm{in}\right)^4}} = 64\,\mathrm{Hz}}.$$

Since the frequency of the plate is less than that of the beam, the fundamental frequency is 64 Hz which is greater than the required 35 Hz minimum.

6.0 SUMMARY

The margins of safety for the G-093 ESS components are all positive. This includes the structural beams, plates, fasteners, and bumpers. The margins of safety are listed in Table 6.0-1 below. All margins of safety are based on an ultimate factor of safety of 2. In addition, the fundamental frequency was found to be greater than 35 Hz. The lowest frequency of the beam mode is 227 Hz and the lowest frequency plate mode is 64 Hz.

Item	**Margin of Safety**			**Page**
	Beam	*Top ESS Fastener*	*Bottom ESS Fastener*	
Beam L1.1	11.6937	5.3887	4.8039	19
Beam L1.2	8.6522	3.7141	4.3826	19
Beam L1.3	8.6522	3.7141	4.3826	19
Beam L1.4	11.6937	5.3887	4.8039	19
Beam L1.5	8.6522	3.7141	4.3826	19
Beam L1.6	8.6522	3.7141	4.3826	19
Beam L2.1	18.8529	4.8039	7.9983	19
Beam L2.2	14.1224	4.3826	7.5614	19
Beam L2.3	14.1224	4.3826	7.5614	19
Beam L3.1	39.1588	7.9983	12.9274	19
Beam L3.2	26.1593	7.5614	12.9274	19
Beam L3.3	26.1593	7.5614	12.9274	19
Beam L23.1	14.1224	4.3826	12.9274	19
Beam L23.2	18.8529	4.8039	12.9274	19
Beam L23.3	14.1224	4.3826	12.9274	19
ESS Plate	1.05			19
Beam Buckling	> 200			20
Computer Fasteners	3.56			21
Fluid Test Cell Fasteners	3.1 (smallest MS)			23
Fluid Reservoir Fasteners	15.21 (smallest MS)			24
Battery Box Fasteners	2.21 (smallest MS)			25
Bumper Clamping Fasteners	0.64 (smallest MS)			25
Bumper Attachment Fasteners	0.446			26
Bumper Viton Padding	0.09			26

Table 6.0-1 Results summary of the G-093 structural analysis