Initial Theoretical Stages in the Development of a Turbulent Radiation–Hydrodynamics Model

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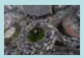
ABSTRACT

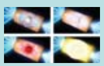
Preliminary stages of this investigation couple the hydrodynamic conservation equations to low-order angular moment radiation transport models. In particular, we consider the Spherical Harmonic (P1) and Maximum Entropy (M1) closures. The second phase of this project is dedicated to the inclusion of turbulent phenomena, by augmenting the radiation hydrodynamics equations by the turbulent kinetic energy and dissipation rate equations. The initial theoretical development and initial numerical investigations are described.

Motivation for Investigation

Shock wave applications occur in a broad variety of applications. Some representative examples where shock waves can occur are traffic flow, supersonic travel, and atmospheric events.

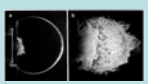


We are particularly interested in the physics of shock wave behavior driven and influenced by radiative effects in natural and controlled environments, such as in astrophysical applications (supernovae and astrophysical jets for example) and in inertial confinement fusion (ICF).

Objectives

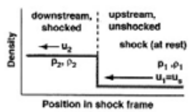
- Develop a closed hydrodynamic model using the Euler equations and the equation of state for a polytropic gas
- Derive radiation transport models using the Spherical Harmonic (P1) and Maximum Entropy (M1) angular closures
- Investigate the effects of turbulence on the developed radiation hydrodynamics models in blast wave applications

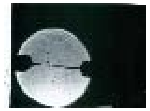


Hydrodynamic Description: Shock Wave Phenomena and the Euler Equations

Shock waves are sudden changes in fluid medium properties.

The *classical Taylor-Sedov* blast wave problem illustrates the evolution of large and instantaneous energy releases from a point source in radially-symmetric systems in non-turbulent environments.





Shock phenomena are mathematically modeled using the Euler equations. The density, momentum, and energy equations, together with the equation of state for a polytropic gas, $e = p/[\rho(\gamma-1)] = c_V T$ are

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{v}) = 0,$$

$$\frac{\partial}{\partial t}(\rho\underline{v}) + \underline{\nabla}\cdot(\rho\underline{v}\underline{v}) + \underline{\nabla}(\gamma-1)\rho\left(E - \frac{\underline{v}\cdot\underline{v}}{2}\right) = 0,$$

$$\frac{\partial}{\partial t}(\rho E) + \underline{\nabla}\cdot\left\{\rho\left[\gamma E - (\gamma-1)\frac{\underline{v}\cdot\underline{v}}{2}\right]\underline{v}\right\} = 0.$$

$\rho \rightarrow$ density
$v \rightarrow$ velocity
$p \rightarrow$ pressure
$e \rightarrow$ specific internal energy
$E = e + \frac{\underline{v}\cdot\underline{v}}{2} \rightarrow$ specific mean total energy

Radiative Description: Radiation Transport in Absorptive Systems and the First Two Angular Moments

Introducing the radiative transport equation in energy regimes where scattering may be neglected (space–time dependence is suppressed),

$$\frac{1}{c}\frac{\partial I(\nu,\underline{\Omega})}{\partial t} + \underline{\Omega}\cdot\underline{\nabla} I(\nu,\underline{\Omega}) = S_A(\nu,\underline{\Omega}) \equiv -\frac{\nu_0}{\nu}\Sigma_A(\nu_0) I(\nu,\underline{\Omega}) + \left(\frac{\nu_0}{\nu}\right)^2 \sigma_A(\nu_0)\, B(\nu_0,T)$$

$$B(\nu_0,T) = \frac{2h\nu_0^3}{c^2}\left(e^{h\nu_0/k_B T} - 1\right)^{-1} \rightarrow \text{Planck function}$$

$$\frac{\nu_0}{\nu} = \gamma_L\left(1 - \frac{\underline{\Omega}\cdot\underline{v}}{c}\right) \rightarrow \text{ratio of CoM to laboratory photon frequencies}$$

$$\gamma_L = \left(1 - \frac{\underline{v}\cdot\underline{v}}{c^2}\right)^{-1/2} \rightarrow \text{Lorentz (relativistic) factor}$$

Taking the zeroth and first moments of the radiative transfer equation and integrating over frequency yields

$$\frac{\partial \Theta}{\partial t} + \underline{\nabla}\cdot\underline{F} = S_{A,0} \equiv -\gamma_L \Sigma_A\left(c\Theta - \frac{\underline{v}\cdot\underline{F}}{c}\right) + \gamma_L^2 \Sigma_A\left(1 + \frac{\underline{v}\cdot\underline{v}}{3c^2}\right) a_r c T^4$$

$$\frac{1}{c^2}\frac{\partial \underline{F}}{\partial t} + \underline{\nabla}\cdot\underline{\underline{P}} = \underline{S}_{A,1} - \gamma_L\frac{\Sigma_A}{c}\left(\underline{F} - \underline{v}\cdot\underline{\underline{P}}\right) - \gamma_L^2\frac{\Sigma_A}{c}\frac{2}{3} a_r \underline{v} T^4$$

** Note that we have an unclosed system of equations. Closures are introduced later.*

Radiation Hydrodynamics: Conservation of Total Energy and Momentum

To develop a radiation hydrodynamics model, we couple the zeroth and first angular moments of the transport equation to the mean energy and momentum equations to ensure conservation of total energy and momentum, respectively,

$$\frac{\partial}{\partial t}(\rho E) + \underline{\nabla}\cdot\left\{\rho\left[\gamma E - (\gamma-1)\frac{\underline{v}\cdot\underline{v}}{2}\right]\underline{v}\right\} = -\frac{\partial \Theta}{\partial t} - \underline{\nabla}\cdot\underline{F} = \gamma_L\Sigma_A\left(c\Theta - \frac{\underline{v}\cdot\underline{F}}{c}\right) - \gamma_L^2\Sigma_A\left(1 + \frac{\underline{v}\cdot\underline{v}}{3c^2}\right) a_r c T^4$$

$$\frac{\partial}{\partial t}(\rho\underline{v}) + \underline{\nabla}\cdot(\rho\underline{v}\underline{v}) + \underline{\nabla}(\gamma-1)\rho\left(E - \frac{\underline{v}\cdot\underline{v}}{2}\right) = -\frac{1}{c^2}\frac{\partial \underline{F}}{\partial t} - \underline{\nabla}\cdot\underline{\underline{P}} = \gamma_L\frac{\Sigma_A}{c}\left(\underline{F} - \underline{v}\cdot\underline{\underline{P}}\right) + \gamma_L^2\frac{\Sigma_A}{c}\frac{2}{3} a_r \underline{v} T^4$$

Eliminating temperature via the polytropic equation of state and combining radiation and hydrodynamics leads to the unclosed system with P1 and M1 closure terms for the divergence of the radiation pressure tensor, respectively,

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{v}) = 0,$$

$$\frac{\partial}{\partial t}(\rho\underline{v}) + \underline{\nabla}\cdot(\rho\underline{v}\underline{v}) + \underline{\nabla}(\gamma-1)\rho\left(E - \frac{\underline{v}\cdot\underline{v}}{2}\right) = -\frac{1}{c^2}\frac{\partial \underline{F}}{\partial t} - \underline{\nabla}\cdot\underline{\underline{P}},$$

$$\frac{\partial}{\partial t}(\rho E) + \underline{\nabla}\cdot\left\{\rho\left[\gamma E - (\gamma-1)\frac{\underline{v}\cdot\underline{v}}{2}\right]\underline{v}\right\} = -\frac{\partial \Theta}{\partial t} - \underline{\nabla}\cdot\underline{F},$$

$$\frac{\partial \Theta}{\partial t} + \underline{\nabla}\cdot\underline{F} = -\gamma_L\Sigma_A\left(c\Theta - \frac{\underline{v}\cdot\underline{F}}{c}\right) + \gamma_L^2\Sigma_A\left(1 + \frac{\underline{v}\cdot\underline{v}}{3c^2}\right) a_r c\left[\frac{1}{c_v}\left(E - \frac{\underline{v}\cdot\underline{v}}{2}\right)\right]^4,$$

$$\frac{1}{c^2}\frac{\partial \underline{F}}{\partial t} + \underline{\nabla}\cdot\underline{\underline{P}} = -\gamma_L\frac{\Sigma_A}{c}\left(\underline{F} - \underline{v}\cdot\underline{\underline{P}}\right) - \gamma_L^2\frac{\Sigma_A}{c}\frac{2}{3} a_r \underline{v}\left[\frac{1}{c_v}\left(E - \frac{\underline{v}\cdot\underline{v}}{2}\right)\right]^4,$$

$$\mathbf{P1}: \underline{\nabla}\cdot\frac{1}{c}\int_{4\pi} d\underline{\Omega}\,\underline{\Omega}\otimes\underline{\Omega}\, I(\underline{r},\underline{\Omega},t) \cong \underline{\nabla}\cdot\frac{1}{c}\int_{4\pi} d\underline{\Omega}\,\underline{\Omega}\otimes\underline{\Omega}\left(\frac{c}{4\pi}\Theta(\underline{r},t) + \frac{3}{4\pi}\underline{\Omega}\cdot\underline{F}(\underline{r},t)\right) = \frac{\underline{\nabla}\Theta(\underline{r},t)}{3}$$

$$\mathbf{M1}: c\underline{\nabla}\cdot\underline{\underline{P}} = \underline{\nabla}\cdot\left(\underline{\underline{D}}\Theta\right),\quad \underline{\underline{D}} = \frac{1-\chi(\underline{f})}{2}\underline{\underline{I}} + \frac{3\chi(\underline{f})-1}{2}\frac{\underline{F}\otimes\underline{F}}{\|\underline{F}\|^2}$$

$$\chi(\underline{f}) = \frac{3 + 4|\underline{f}|^2}{5 + 2\sqrt{4 - 3|\underline{f}|^2}} \qquad \underline{f} = \frac{\underline{F}}{c\Theta}$$

Turbulence

Turbulent fluid motion is an irregular flow condition in which quantities have random space–time variations and exhibit stochastic effects.

One-equation models based on the turbulent kinetic energy equation have limitations as they heuristically relate the turbulent lengthscale to a characteristic flow dimension (a mixing-length).

Two-equation models, such as the $K-\varepsilon$ model are based on an eddy viscosity, provide a turbulent lengthscale, and are more complete.

Initial stages on adopting a $K-\varepsilon$ model have been applied to the *classical Taylor-Sedov* blast wave problem. The Reynolds-averaged Navier–Stokes equations are

$$\left[\frac{d}{dt} + \left(\frac{\partial}{\partial r} + \frac{d-1}{r}\right)\overline{\overline{v}}\right]\begin{pmatrix}\overline{\rho}\\ \overline{\rho}\overline{\overline{v}}\\ \overline{\rho}\overline{\overline{E}}\\ \overline{\rho}K\\ \overline{\rho}\varepsilon\end{pmatrix} = \begin{pmatrix}0\\ -\frac{1}{r^{d-1}}\frac{\partial}{\partial r}\left(r^{d-1}\tau\right) - \frac{\partial \overline{p}}{\partial r}\\ -\overline{p}\left(\frac{\partial}{\partial r} + \frac{d-1}{r}\right)\left(\overline{\overline{v}} + \frac{\nu_t}{\sigma_\rho\overline{\rho}}\frac{\partial\overline{\rho}}{\partial r}\right) + D_K - \Pi_K + T_U\\ P_K^b + P_K^s - D_K + \Pi_K + T_K\\ \frac{\varepsilon}{K}\left(C_{\varepsilon 0}P_K^b + C_{\varepsilon 1}P_K^s + C_{\varepsilon 2}D_K + C_{\varepsilon 3}\Pi_K\right) + T_\varepsilon\end{pmatrix}$$

** Highlighted terms signify turbulence contributions.*

In developing a turbulent radiation–hydrodynamics model, cross sections need be expressed in terms of hydrodynamic variables to correctly perform Reynolds averaging,

$$\Sigma_A \propto \frac{\rho}{T^3} \cong \frac{\xi\rho}{T^3}.$$

Ongoing and Future Investigations

- Extend eigenstructure studies to the *non-turbulent* P1 and M1 radiation hydrodynamics models
- Develop a Riemann solver for the *non-turbulent* and hyperbolic portions of *turbulent* model equations
- Further investigate *turbulent* phenomena (including self-similar analyses) and develop boundary conditions and Rankine-Hugoniot jump conditions
- Generate a computationally feasible *turbulent* radiation hydrodynamics model in which both hyperbolic and parabolic characteristics are correctly treated

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